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Title: The beam envelope equation and practical applications

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# The Beam Envelope Equation and Practical Applications

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J-5 Physics Colloquium

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## Today, I will try to answer some questions that you might have about the beam envelope equation;

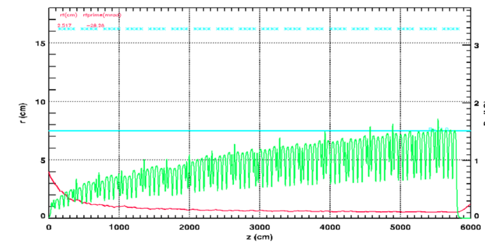
- What is it?
- How is it derived?
- What are its advantages?
- What are its limitations?
- How can it be applied to practical problems?
- How well does it agree with experiments?

# What is it?

- **There are many beam properties of interest.**
  - Size, position, motion, energy, momentum,emittance,…
  - All are of practical interest for radiographic spot size, as well as practical concerns such as beam spill and associated backgrounds.
  - Computer codes have significantly contributed to our understanding.
- **There are many theoretical ways of addressing these properties.**
  - Matrix optics to track the beam through discrete elements like magnets;
    - Codes: TRANSPORT, TRACE-3D, REVMOC, …
  - Particle tracking through distributed fields;
    - Codes: Egun, Trak, GPT, …
  - Particle in cell (PIC) codes that explicitly calculate all forces on electrons
    - Codes: MAGIC, LSP, WARP, VORPAL, …
  - Envelope equations that calculate average forces on electrons
    - Codes: Scrape, XTR, LAMDA, …
- **The envelope equation explicitly addresses the evolution of the <u>beam size</u> as it is transported through the accelerator and to the bremsstrahlung target to produce the radiographic source spot.**

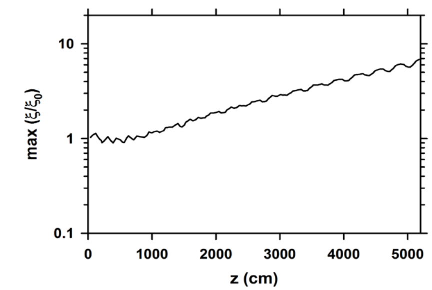
# We use many of these methods to better understand beam dynamics in the DARHT accelerators, and also to predict the performance of advanced accelerators.

- **Beam Production**
  - Tricomp **Trak** orbit tracking code
  - **LSP** Particle-in-cell (PIC) code



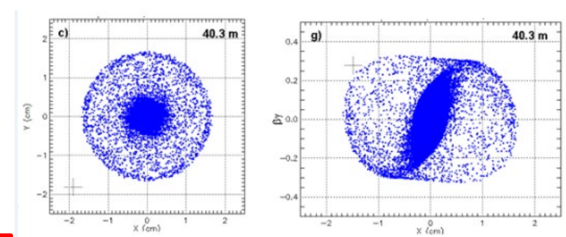

**XTR** simulation of matched transport

- **Accelerated-Beam Transport through the LIA**
  - **XTR** static envelope and centroid code
    - Includes Image Displacement Instability algorithm
  - **LAMDA** Time resolved envelope and centroid code
    - Includes Beam-Breakup (BBU), Ion-Hose and Resistive-Wall Instability algorithms
  - **LSP-Slice** PIC code




**LAMDA** simulation of BBU growth

- **Coasting-Beam Transport from LIA to Target**
  - **LAMDA** Time resolved envelope and centroid
    - Includes Image Displacement Instability algorithm
  - **LSP-Slice** PIC code




**LSP** PIC simulation of beam halo

**Our "go-to" codes for most experimental applications are XTR and LAMDA**

# How is it derived?

- It all begins with the equation of motion for a single electron.

$$\frac{d\mathbf{p}}{dt} = -e\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right] \quad \text{(SI)}$$

  - In an LIA, the fields can be external (due to gaps and magnets) as well as internal (due to space charge and beam current)
- Solve equations of motion with a few presuppositions:
  - **Azimuthal symmetry (round beam)**
  - External Bz, Ez uniform within beam
  - Paraxial approximation, trajectory angles with axis are small, $\mathbf{v}_\perp << \mathbf{v}_\mathbf{z}$
- Average and take moments.

$$R_{rms}^2 = \frac{1}{N}\sum_{i=1}^{N} r_i^2$$

  - etc.
  - For details see E. Lee and R. Cooper, Part. Accel. Vol 7,1976, pp. 83-95
  - One more little approximation:
    - for relativistic beams $v_z=\beta c$ is almost constant, so $d/dz \approx 1/\beta c \; d/dt$

# The equation derived from all of this is known as the "envelope equation."

$$\boxed{\frac{d^2 R_{env}}{dz^2} + \frac{1}{\beta^2\gamma}\frac{d\gamma}{dz}\frac{dR_{env}}{dz} + \left[\frac{1}{2\beta^2\gamma}\frac{d^2\gamma}{dz^2} + k_\beta^2\right] R_{env} = \frac{K}{R_{env}} + \frac{\varepsilon_n^2 + \left(P_\theta / m_e c\right)^2}{\left(\beta\gamma\right)^2 R_{env}^3}}$$

$R_{env} = \sqrt{2}\, R_{rms}$ $\quad$ $\beta = v_z / c$ $\quad$ $\gamma = 1/\sqrt{1-\beta^2}$ $\quad$ $KE = (\gamma - 1) m_e c^2$

$$k_\beta = \frac{eB}{2\beta\gamma m_e c} = \frac{B_{kG}}{3.41\beta\gamma} \ \text{cm}^{-1}$$ (betatron wavenumber)

$$K = \frac{2I_b}{\beta^2 I_A}\left[\left(1 - f_e\right) - \beta^2\left(1 - f_m\right)\right] \Rightarrow \frac{2I_b}{\beta^2\gamma^2 I_A}$$ (generalized perveance)

$$I_A = \frac{4\pi}{\mu_0}\frac{m_e c}{e}\beta\gamma = 17.05\text{kA}\beta\gamma$$ (Alfven limiting current)

$$P_\theta = \gamma m_e R_{env}\omega - eR_{env}A_\theta$$ (canonical angular momentum)

# The terms of the envelope equation have physical meaning.

$$\frac{d^2R_{env}}{dz^2}+\overbrace{\frac{1}{\beta^2\gamma}\frac{d\gamma}{dz}\frac{dR_{env}}{dz}+\left[\frac{1}{2\beta^2\gamma}\frac{d^2\gamma}{dz^2}+k_\beta^2\right]}^{\text{External Forces}}R_{env}=\overbrace{\frac{K}{R_{env}}+\frac{\varepsilon_n^2+\left(P_\theta/m_ec\right)^2}{\left(\beta\gamma\right)^2R_{env}^3}}^{\text{Internal Forces (Defocusing)}}$$

Adiabatic damping by axial electric field

Focusing by radial electric field

Focusing by solenoidal magnetic field

Centripetal force

Emittance (“temperature”)

Space-charge force

(defocusing in LIA vacuum)

# The space-charge term deserves its very own vugraph, because it can be either defocusing (in vacuum) or focusing (in gas or plasma).

- **Internal space-charge forces are governed by the generalized perveance, which takes into account beam neutralization in a gas or plasma:**

$$K = \frac{2I_b}{\beta^2 I_A}\left[\left(1-f_e\right)-\beta^2\left(1-f_m\right)\right]$$

- **Neutralization of the beam space charge by background ions is quantified by the electrical neutralization factor $f_e = N_i / N_e$**
- **Neutralization of the beam current by an induced return current is quantified by the magnetic neutralization factor $f_m = I_r / I_b$**
- **In vacuum $f_e = f_m = 0$ and the space-charge force is defocusing;**

$$K = \frac{2I_b}{\beta^2\gamma^2 I_A} = \frac{2I_{kA}}{17.05\beta^3\gamma^3}$$

- **If $f_m = 0$ in gas or plasma, then**

$$K = \frac{2I_b}{\beta^2 I_A}\left[\left(1-\beta^2\right)-f_e\right] = \frac{2I_b}{\beta^2 I_A}\left[\frac{1}{\gamma^2}-f_e\right]$$

- **And if $f_e > 1/\gamma^2$ the space-charge force switches to focusing. Known as Ion-Focused Regime (IFR)**

# The emittance also deserves a page of its own, because of differing definitions.

- **The emittance $\varepsilon_n$ in this equation is the normalized Lapostolle emittance, sometimes called the "4-rms" emittance.**

$$\varepsilon_n = 2\beta\gamma\sqrt{\left\langle r^2\right\rangle\left[\left\langle r'^2\right\rangle + \left\langle\left(v_\theta / \beta c\right)^2\right\rangle\right] - \left\langle rr'\right\rangle^2 - \left\langle rv_\theta / \beta c\right\rangle^2}$$

- **For a rigidly rotating beam this reduces to the more familiar form**

$$\varepsilon_n = 2\beta\gamma\sqrt{\left\langle r^2\right\rangle\left\langle r'^2\right\rangle - \left\langle rr'\right\rangle^2}$$

- **The normalized emittance is invariant in the absence of nonlinear transverse forces.**
  - **Near the axis nonlinear external fields are small**
  - **For a uniform beam, the space-charge forces are linear**
  - **Therefore, for small uniform beams, $\varepsilon_n$ is very nearly invariant**

- **Since <*r*²> = <*x*²+*y*²> = <*x*²>+<*y*²>=2<*x*²>, etc, one has** $\varepsilon_n = 4\beta\gamma\sqrt{\left\langle x^2\right\rangle\left\langle x'^2\right\rangle - \left\langle xx'\right\rangle^2}$

# Canonical angular momentum also deserves its own page.

- **Canonical angular momentum is sum of mechanical and field momenta: $P_\theta$ = ?**
  - **There is no mechanical angular momentum at the cathode**
    - **Angular velocity of emitted electrons averages out.**
  - **A bucking coil is used to null the field momentum at the cathode. ("shielded cathode")**
    - **Field momentum is proportional to flux linking the cathode, so one adjusts bucking for** $$\Phi = \int B_z 2\pi r dr = 0$$
    - **=> $P_\theta$ = 0 for shielded cathodes in our LIAs.**
- **Busch's theorem states that the canonical angular momentum is invariant in cylindrically symmetric magnetic fields,.**
- **In our LIAs, we can usually neglect the angular momentum term on the RHS of the envelope equation.**
  - **The beam rotates in the axial magnetic field to compensate for the field momentum, and the sum of the two is zero.**

# Summarizing:

- **Normalized emittance $\varepsilon_n$ is invariant if transverse forces are linear in *r*.**
  - Fringe fields of solenoids have radial components that are increasingly nonlinear away from axis
  - Design LIAs to have minimal fringe fields (e.g., long solenoids)
  - Design magnetic tunes for small matched beam size
- **Canonical angular momentum is invariant in cylindrical symmetry and zero for shielded cathode. $\therefore$ For minimum spot size at target:**
  - Use solenoids for transport, not quads.
  - Design solenoids to have minimal multipole aberrations.
- **In a well-designed LIA with optimal tune, the RHS of envelope equation depends on *z* only through $R(z)$ and $\beta\gamma(z)$**

# The envelope equation can be simplified by eliminating the $R'_{env}$ term.

- **Transforming to $r = (\beta\gamma)^{1/2} R_{env}$ will eliminate the first derivative term, and the transformed equation is**

$$\frac{d^2 r}{dz^2} + k_r^2 r = \frac{\beta\gamma K}{r} + \frac{\varepsilon_n^2}{r^3}$$

$$k_r^2 = k_e^2 + k_\beta^2$$

$$k_e = \frac{\gamma \gamma'}{4(\beta\gamma)^2} \qquad \{ \propto E(z)/\gamma \}$$

- **This form is useful anywhere that there are axial and radial electric fields, e.g. in diode, through gap region, or near grounded target.**
- **For emittance dominated beam with $k_r \approx$ constant, one has $r \approx$ constant, and $R_{env} \approx$ constant/ $(\beta\gamma)^{1/2}$, which is called adiabatic damping.**
- **All transverse motion (e.g., BBU) is adiabatically damped by axial acceleration.**

## The envelope equation is even simpler if the beam energy is constant. (This is commonly called a "coasting beam.")

- **Simplified envelope equation:**

$$\frac{d^2R}{dz^2} + k_\beta^2 R = \frac{K}{R} + \frac{\varepsilon^2}{R^3}$$

$$R \equiv R_{env} = \sqrt{2}\ R_{rms}$$

$$\varepsilon = \varepsilon_n / \beta\gamma$$

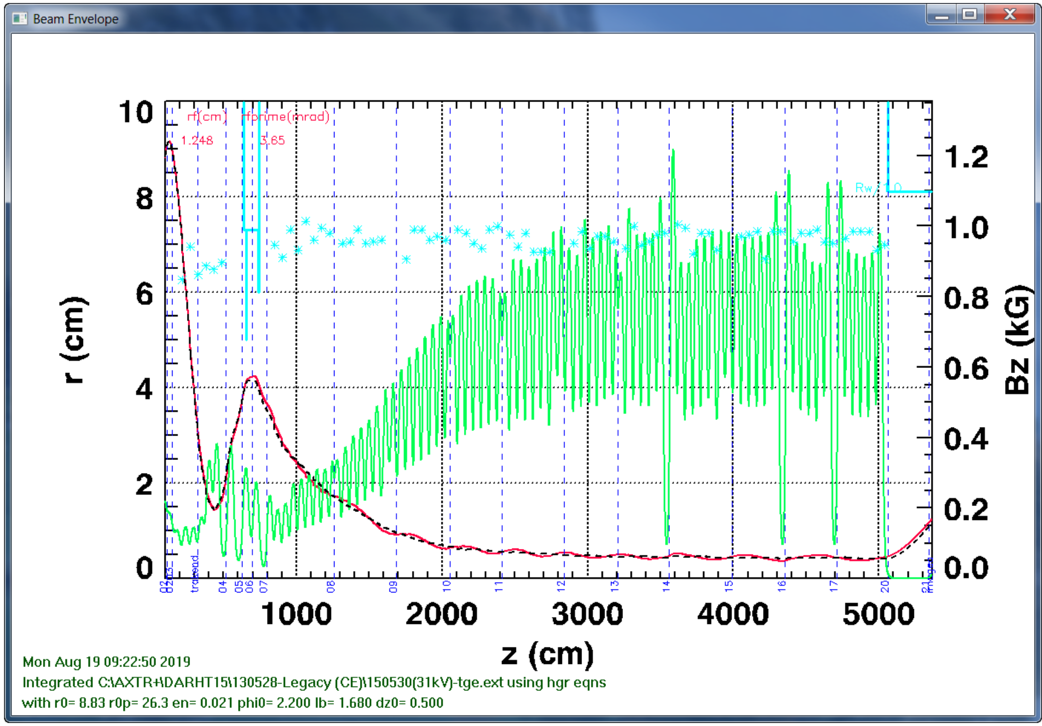
- **Useful in regions with no external electric fields, e. g. in between accelerating gaps, downstream transport, final focus, and simple physics models.**
- **Lends itself to rapid numerical solutions.**

# Our XTR and LAMDA envelope codes solve the simplified equation:

- **Envelope equation is second order:**
  - Numerically solve for variables $R$ and $R' = dR/dz$ with given initial conditions $R_0$ and $R_0'$
  - XTR solver: High resolution matrix optics
  - LAMDA solver: 4$^{th}$ order Runge-Kutta
- **Ansatz for electric fields in an LIA:**
  - Gaps are treated as thin lenses that
    1. Focus beam envelope
    2. Add an increment of energy
- **Include second order corrections for space charge and diamagnetism.**
  - Space charge; at beam edge in pipe with radius $b$; $e\Delta\phi \approx 60\ I_b \ln(b/R) \sim 100 - 200$ keV
  - Diamagnetism; beam rotation increases $B_z$ at edge; $\Delta B/B \approx (R^2/b^2)\ I_b\ /\ 2I_0\beta\gamma < 1\%$
- **XTR gives a time-independent solution**
- **LAMDA gives a time-resolved solution**
  - Beam is segmented into disks
  - Disks don't communicate with neighbors (transverse dynamics dominate)
  - Envelope Equation solved for each disk

# Differences in second order corrections introduce slight differences in results for DARHT from XTR and LAMDA.

- **XTR: Second order corrections for diamagnetism and space charge**
  - **"Beam dynamics equations for xtr," Paul Allison, LA-UR-01-6585, 2001**
- **LAMDA: Second order corrections for diamagnetism, space charge, and $R_0'$**
  - **"Improved envelope and centroid equations for high-current beams," Tom Genoni, Tom Hughes, and Carsten Thoma, *Proc. 14th Beams Conf.*, 2002**




**DARHT-II nominal tune. Red: XTR; Black: LAMDA**

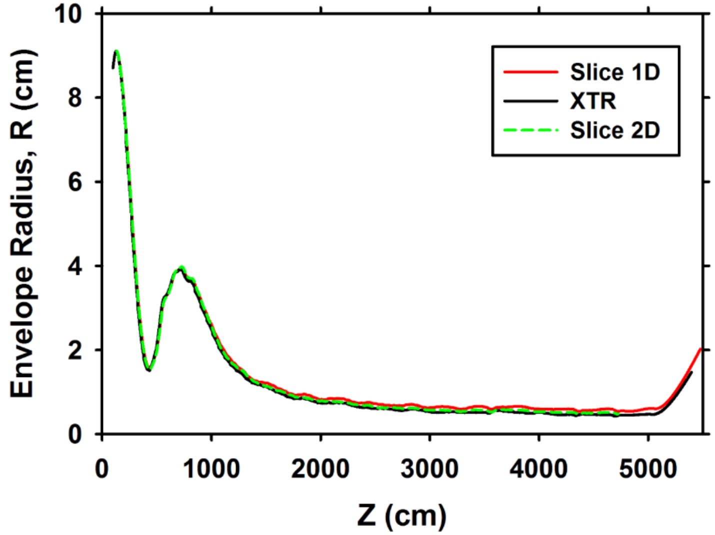
# Comparison with other approaches to beam transport, e. g. Matrix Optics, or PIC simulations.

- **Matrix Optics: As in TRANSPORT code.**
- **Envelope: As in LAMDA**
- **PIC: Slice or Full 4-D**

| | **Speed** | **External Forces** | **Internal Forces** | **R(z)** | **$\varepsilon$(z)** | **Transverse Density** | **Phase Space** | **Time Resolved** |
|---|---|---|---|---|---|---|---|---|
| **Matrix** | fastest | Discrete | No | discrete | constant | moments | moments | no |
| **Envelope** | fast | Averaged | Moments | resolved | modeled | rms | $\varepsilon = A/\pi$ | yes |
| **PIC** | slowest | Mapped | Detailed | resolved | resolved | resolved | resolved | yes |

# Envelope code results compare favorably with PIC and ray-trace code results.

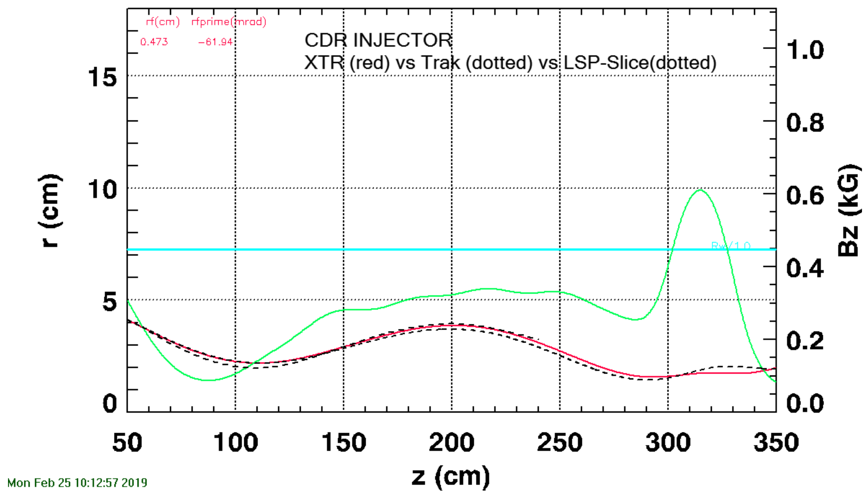
## DARHT-II code comparison




**Comparison of envelope code (XTR) with PIC code (LSP).**

**Red: PIC-Slice (LSP) in cylindrical coordinates**
**Green: PIC-Slice (LSP) in cylindrical coordinates.**
**Black: Envelope (XTR)**

## Scorpius CDR Injector code comparison




**Comparison of envelope code (XTR) with PIC (LSP) and ray-trace (TRAK).**

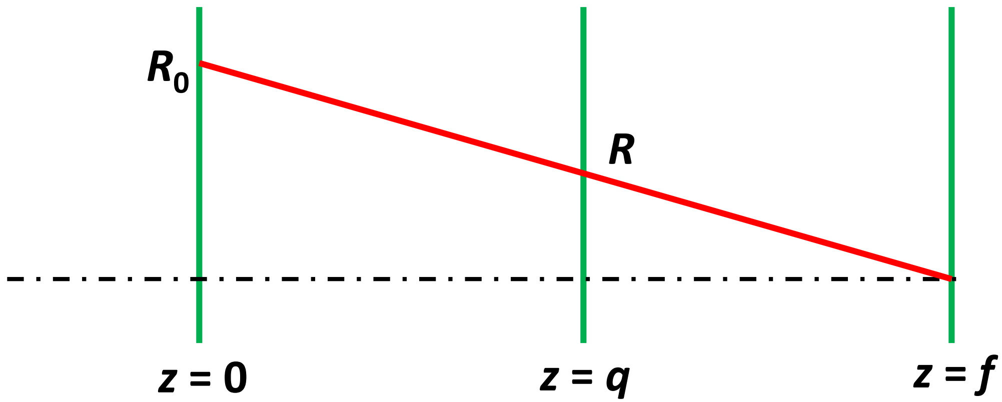
**Green: Solenoidal magnetic field on axis.**
**Cyan: Beam Tube.**
**Red: Beam envelope (XTR).**
**Dashed Black: Ray trace (TRAK).**
**Dashed Black to exit: PIC (LSP)**

# HERE ARE A FEW APPLICATIONS:

- **Coasting and drifting beam ($k_\beta = 0$, $\gamma$ = constant)**
  - Emittance Dominated
  - Space charge dominated
- **Coasting through magnetic field ($k_\beta \neq 0$, $\gamma$ = constant)**
  - Bz = constant
  - Bz = periodic
- **Linear Induction Accelerator**
  - Tuning
  - Measuring emittance with solenoid scan technique

# Simple Application: The envelope equation for a coasting beam can be used to find the size of an emittance-dominated beam on target.

- **Envelope equation:** $$\frac{d^2R}{dz^2} = \frac{\varepsilon^2}{R^3}$$
- **Solution:** $$R^2 = R_0^2 + 2R_0R_0'z + \left(\varepsilon^2 / R_0^2 + R_0'^2\right)z^2$$
- **Define $f = -R_0/R_0'$**
- **Target location is at $z = q$,**




- **Rearrange solution:**

$$\boxed{R^2 = (1 - q/f)^2 R_0^2 + (\varepsilon q / R_0)^2}$$

# Another Example: The equation for a coasting beam can also be used to estimate the chromatic aberration of final focus solenoid

$$R^2 = (1 - q/f)^2 R_0^2 + (\varepsilon q / R_0)^2$$

- Smallest spot when $f = q$, giving $R = \varepsilon q / R_0$
- Blur due to $df/d\gamma$ can be estimated by finding $R$ when $\varepsilon = 0$
- $f \propto p^2$ for solenoids, where $p = \beta\gamma m_e c$
- Differentiate $R^2$ wrt $\gamma$ using $df/d\gamma = (df/dp)(dp/d\gamma) = 2\gamma$
- Result* : $dR = (q/f)\, 2R_0\, d\gamma / \gamma$
- In particular, at focus, with $q = f$ : $\boxed{dR = 2R_0\, d\gamma / \gamma}$
  - (Same result as the confusing “circle of least confusion” estimation)
- Blurred spot size at focus is

$$R^2 \approx 4R_0^2 (d\gamma / \gamma)^2 + (\varepsilon q / R_0)^2$$

- NB: The Result* applies to any, and all, $R(f)$, so it can be used to estimate <u>uncertainty in a scan</u> (obtained by varying $f$) due to beam energy spread and chromatic aberration of the final focus solenoid.

# Alternate derivation of chromatic aberration for an emittance dominated beam. Use moments of the distribution of energy variation, $\gamma$ => $\gamma$ + $\delta\gamma$.

$$R^2 = (1 - q/f)^2 R_0^2 + (\varepsilon q / R_0)^2$$
$$f = f + \delta f_i$$
$$|\delta f_i| << 1, \quad \langle \delta f_i \rangle = 0$$
$$\delta f_i = \delta\gamma_i \frac{df}{d\gamma} = \delta\gamma_i \frac{df}{dp}\frac{dp}{d\gamma} = \delta\gamma_i \left(2\frac{f}{p}\right)\left(\frac{\gamma}{p}\right) = 2f\frac{\delta\gamma_i}{\beta^2\gamma}$$
$$|\gamma_i| << 1, \quad \langle \gamma_i \rangle = 0$$
$$(1 - q/f)^2 = \left(1 - \frac{q}{f + \delta f_i}\right)^2 = \left(1 - \frac{q}{f}\left(1 - \frac{\delta f_i}{f} + \cdots\right)\right)^2$$
$$\approx (1 - q/f)^2 - 2(1 - q/f)\frac{\delta f_i}{f} + \left(\frac{\delta f_i}{f}\right)^2$$

$$\langle g_i^p \rangle \equiv \frac{1}{N}\sum_{i=1}^{N} g_i^p$$

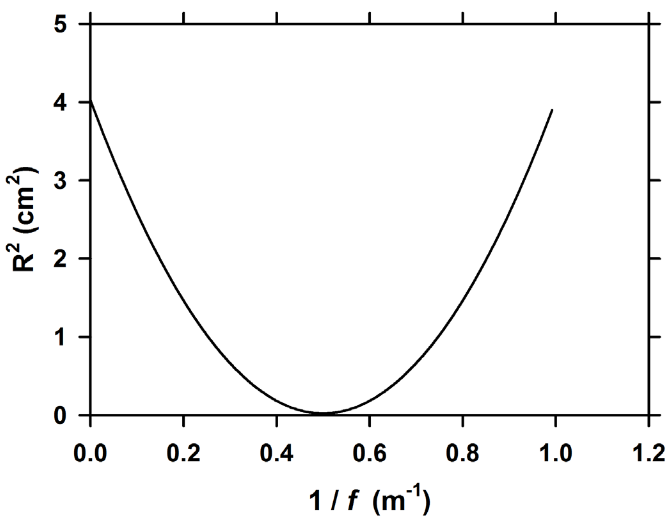
$$\langle R^2 \rangle = \left\langle (1 - q/f)^2 R_0^2 + (\varepsilon q / R_0)^2 \right\rangle$$
$$= \left\langle (1 - q/f)^2 \right\rangle R_0^2 + (\varepsilon q / R_0)^2$$
$$= (1 - q/f)^2 R_0^2 - 2(1 - q/f)\frac{1}{f}\langle \delta f_i \rangle R_0^2 + \frac{1}{f^2}\langle \delta f_i^2 \rangle R_0^2 + (\varepsilon q / R_0)^2$$
$$= R^2 + \left(2\frac{R_0}{\beta^2\gamma}\right)^2 \langle \delta\gamma_i^2 \rangle = \boxed{R^2 + \left(2\frac{R_0 \delta\gamma_{rms}}{\beta^2\gamma}\right)^2}$$

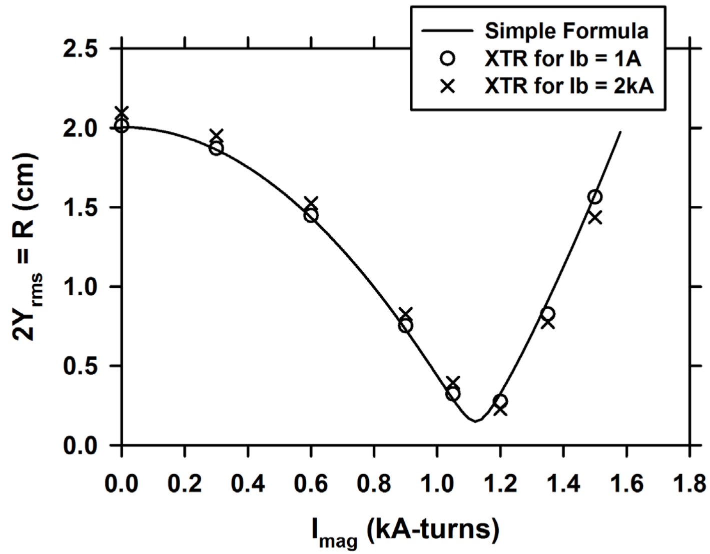
# Application: Focusing magnet scan of an emittance dominated beam (N.B. high current beams are not emittance dominated)

- **Solution:** $$R^2 = (1 - q/f)^2 R_0^2 + (\varepsilon q / R_0)^2$$
- **Thin lens: $f \propto (\gamma / B)^2$**
- **Produces characteristic curves for a scan of focusing strength.**

**Example:**

- $KE$ = 20 MeV
- $I_b$ = 1A or 2kA
- $\varepsilon_n$ = 0.060 cm-rad
- Solenoid:
  - $L_{eff}$ = 10 cm
  - $R_{eff}$ = 9.612 cm
  - (thin-lens) $f$ = 2.517m @ 1.0 kA-turn



# Application: Matched transport

- **Consider a beam coasting through a uniform guide field:**

$$\frac{d^2 R}{dz^2} + k_\beta^2 R = \frac{K}{R} + \frac{\varepsilon^2}{R^3}$$

- **Constant envelope radius => Matched transport;** $\boldsymbol{R \equiv R_m}$

$$k_\beta^2 = \frac{K}{R_m^2} + \frac{\varepsilon^2}{R_m^4}$$

- **Solution:**

$$R_m^2 = \frac{1}{2k_\beta^2}\left[K + \sqrt{K^2 + 4k_\beta^2 \varepsilon^2}\,\right]$$

- **Space Charge dominated :** $\boldsymbol{R_m = K^{1/2} / k_\beta}$ **; Scaling:** $\boldsymbol{(I_b / \beta\gamma)^{1/2} / B}$

- **Emittance dominated:** $\boldsymbol{R_m = (\varepsilon/k_\beta)^{1/2}}$ **; Scaling :** $\boldsymbol{(\varepsilon_n/B)^{1/2}}$

# Application: Stability of Envelope Oscillations in Periodic Focusing

- **Consider small perturbations on the envelope of a matched beam coasting through a periodic guide field, $B = B_0[1 - \Delta\cos(2\pi z/L)]$**

$$\frac{d^2\delta R}{dz^2} + k^2\delta R = 0$$

- **Can be transformed to Mathieu Equation by change of variable,**

$$\frac{d^2\psi}{d\zeta^2} + (a - 2q\cos 2\zeta)\psi = 0$$

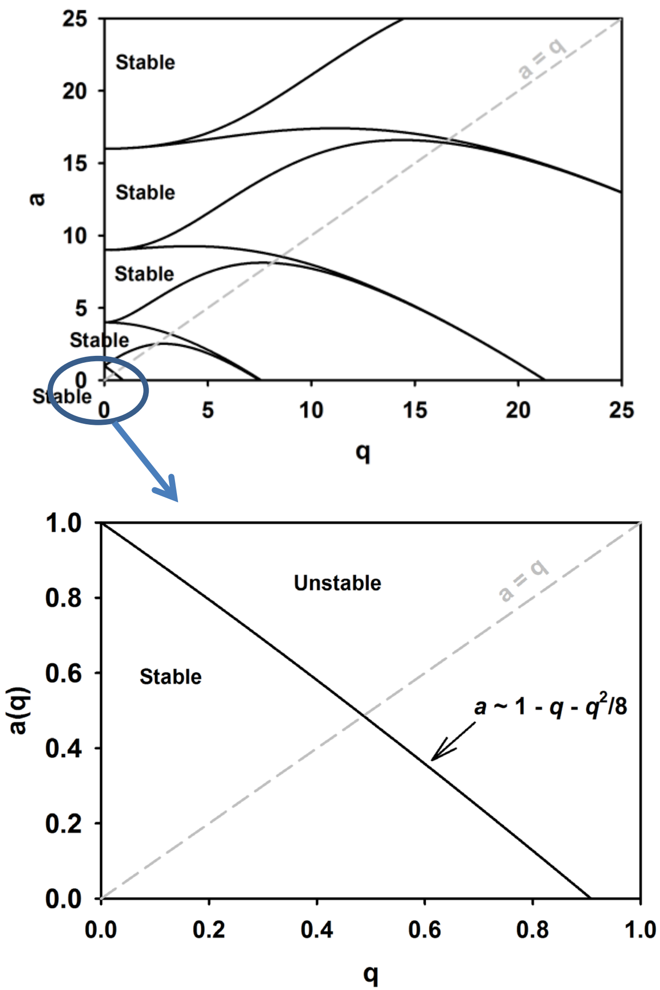
- **with**

$$a = \left(\frac{k_0 L}{\pi}\right)^2 = \left(\frac{2k_{\beta 0}L}{\pi}\right)^2 - 2\frac{K}{r_m^2}\left(\frac{L}{\pi}\right)^2$$

$$q = \left(\frac{2k_{\beta 0}L}{\pi}\right)^2 \frac{\delta B}{B_0}$$

# More: Stability of Envelope Oscillations in Periodic Focusing

- **Mathieu equation has well-known stability properties:**




**From envelope equation:**

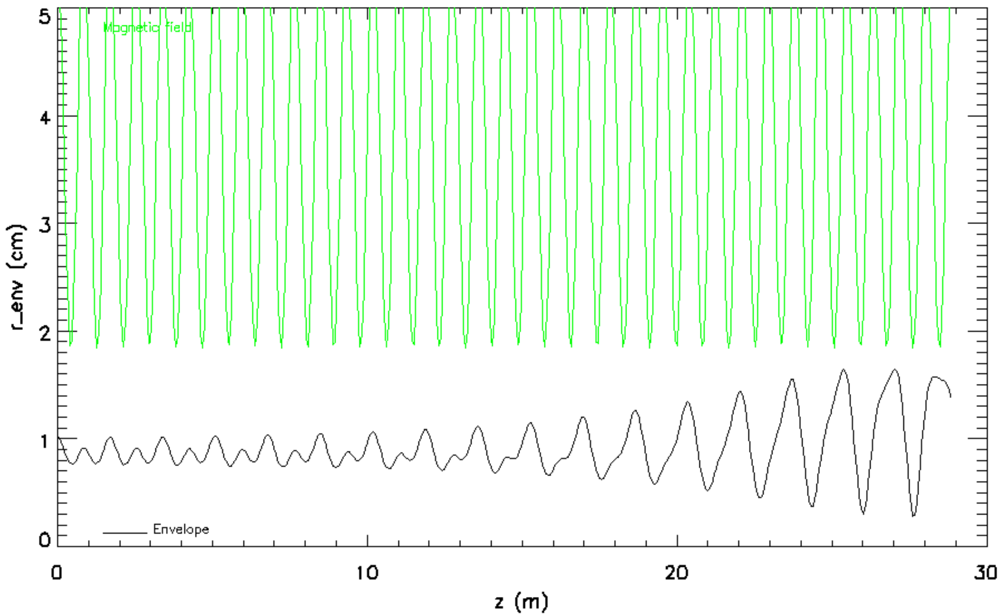
$$k_\beta L < \frac{\pi}{2}\left[1+2\frac{K}{r_m^2}\left(\frac{L}{\pi}\right)^2\right]^{1/2}\left(1+\frac{\delta B}{B}\right)^{-1/2}$$

**For DARHT-Like parameters, stability generally requires phase-advance/cell**

$$k_\beta L < \pi / 2$$

# Even More: Envelope instability can be a problem for LIAs needing high magnetic fields to suppress BBU at low energy.

- Scorpius-like parameters in low energy section:



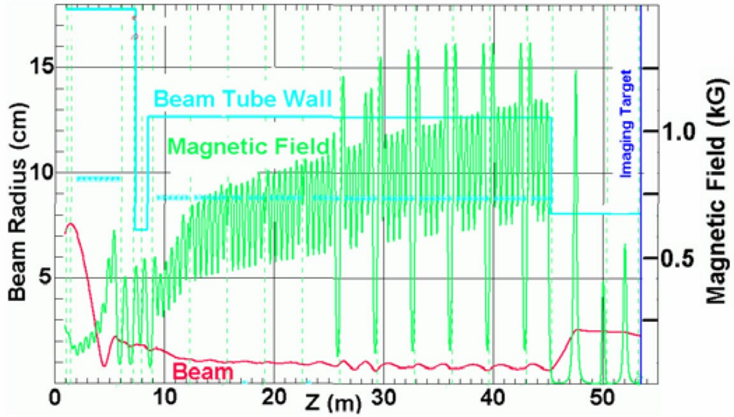

Full-envelope solution shown in black for periodic field shown in green.

Parameters:

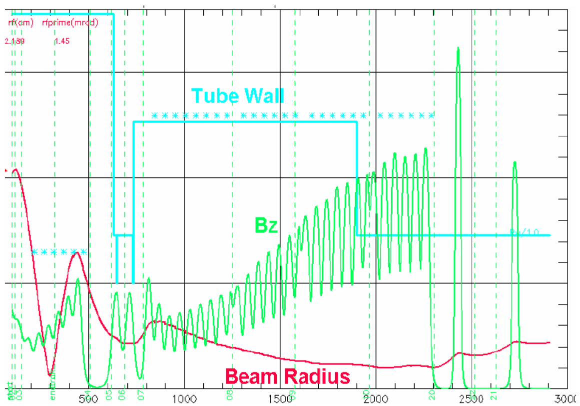
- $KE$ = 4 MeV
- $I_b$ = 2 kA
- $\varepsilon_n$ = 600 mm-mr
- $B_{av}$ =650 G
- $L_{cell}$ = 85 cm

# Application: XTR has been used to design all of the tunes for the DARHT LIAs for 20 years. DARHT-II was tuned and retuned for 4 different configurations.

**2002-2003, First Beam, 72 cells**

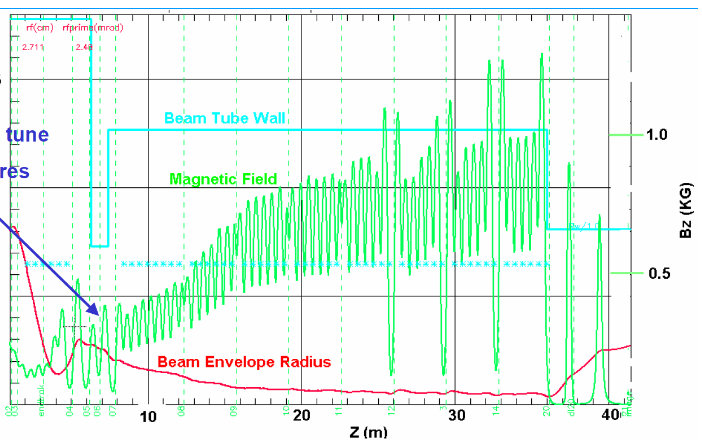



**2006-2007, Scaled Accelerator, 32 cells**

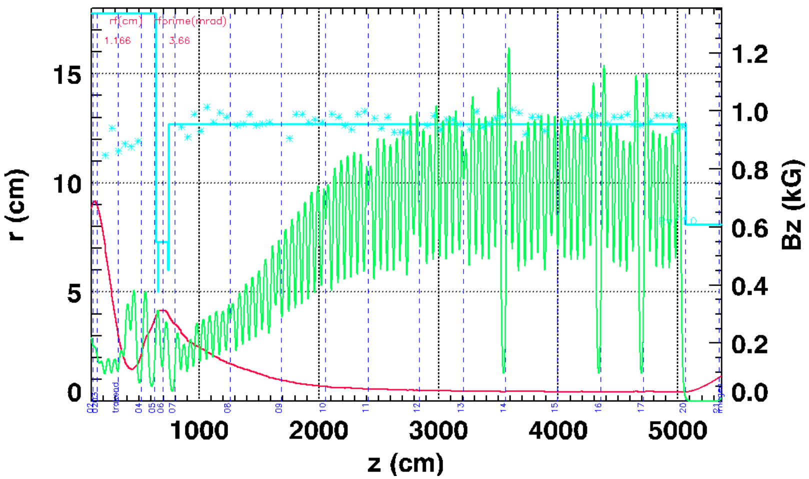



**2005, Stability Tests, 56 cells**

BCUZ:
- "Smooth" tune
- No apertures



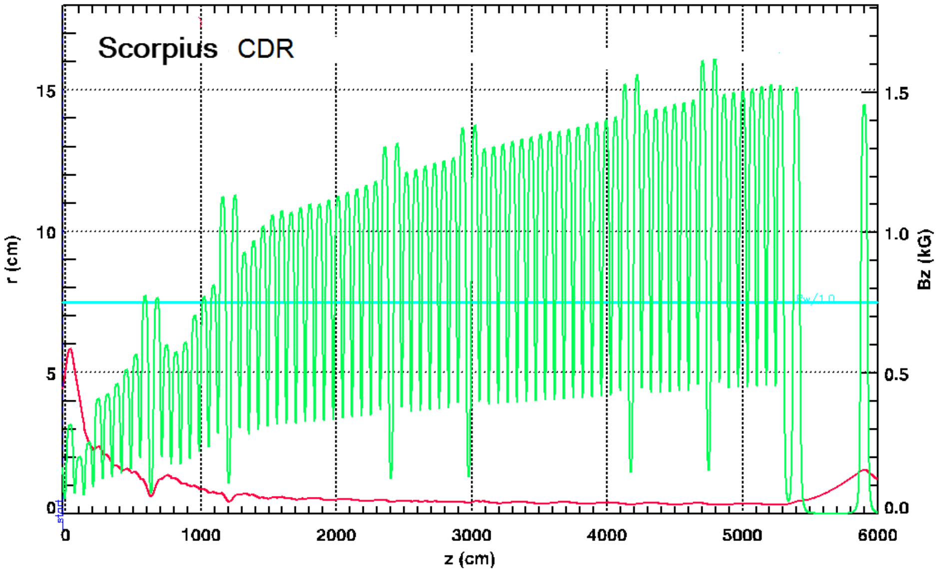

**2007-Present, Final Configuration, 74 cells**



# Application: XTR and LAMDA have been used to design tunes for new accelerators like ARIA and Scorpius.

**A Magnetic Transport Tune for the Scorpius Conceptual Design Report (CDR)**

- **The first cellblocks were used to focus the beam down to a small size to prevent emittance growth from accumulated spherical aberration of solenoids.**
- **The magnetic field in the first cell block is high enough to prevent the Image Displacement Instability (IDI).**
- **Magnetic field is high enough to suppress BBU, but without excessive magnet heating.**
- **Magnetic field increases approximately as $\sqrt{\gamma - \gamma_0}$ to minimize phase advance in order to reduce corkscrew motion.**



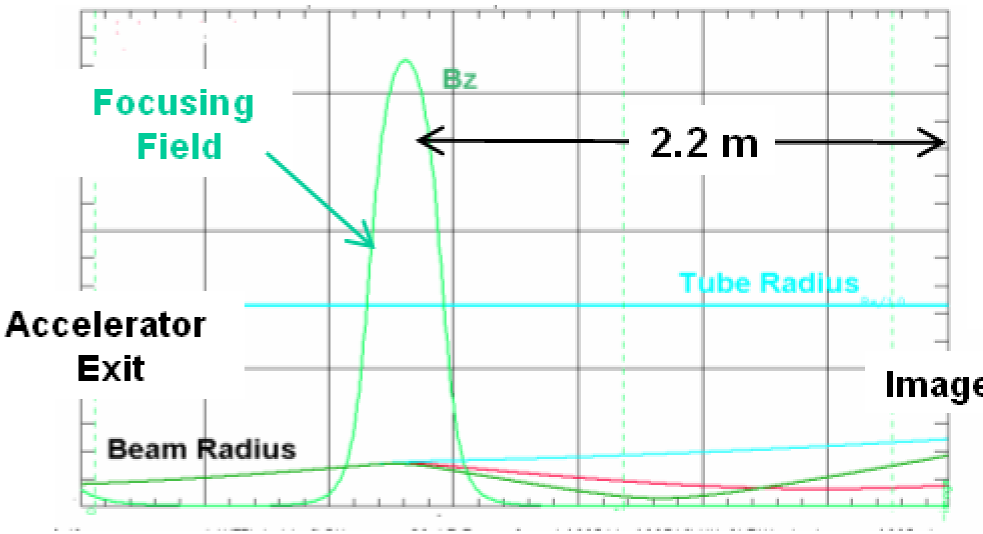

**Initial beam parameters:**

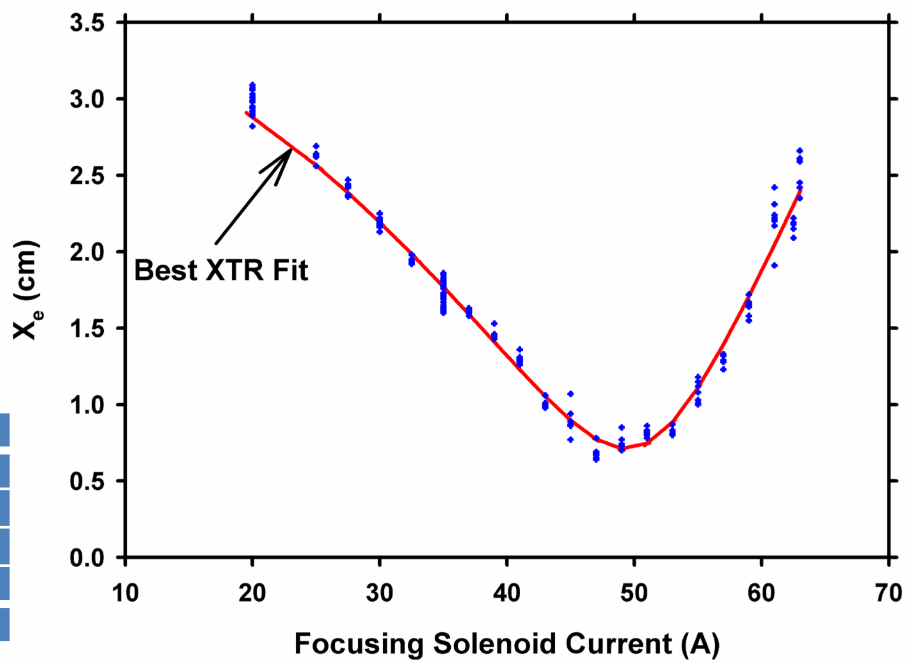
- **Injected Energy, $KE_0$ = 2.0 MeV**
- **Beam Current, $I_b$ = 2.0 kA**
- **Initial envelope radius, $r_0$ = 4.95 cm**
- **Initial divergence, $r_0'$ = 42 mr**
- **Normalized emittance, $\varepsilon_n$ = 305 mm-mr**

# Application: Emittance scan

**Proper experimental design of scan measurement to minimize uncertainty:**

- Distance between focusing element and target must be great enough that target is not damaged at minimum size.
- Focusing element must have enough strength to generate complete curve.
- Need lots of data for statistically significant fit
  - Imaging should collect lots of data => streak camera
  - Only need single view (Vertical), because $2y_{rms} = R_{env}$
  - Anamorphic optics (slitless) simplifies alignment




## DARHT-II 8-MeV Scaled LIA

| Parameter | Value | Uncertainty |
|---|---|---|
| Energy | 8.0 MeV | ± 0.5% absolute |
| Current | 0.9 to 1.1 kA | ± 2% |
| Envelope size | 0.8 cm | ± 3% |
| Envelope divergence | 3.2 mrad | ± 16% |
| Emittance (normalized) | 617 p(mm-mrad) | ± 10% |

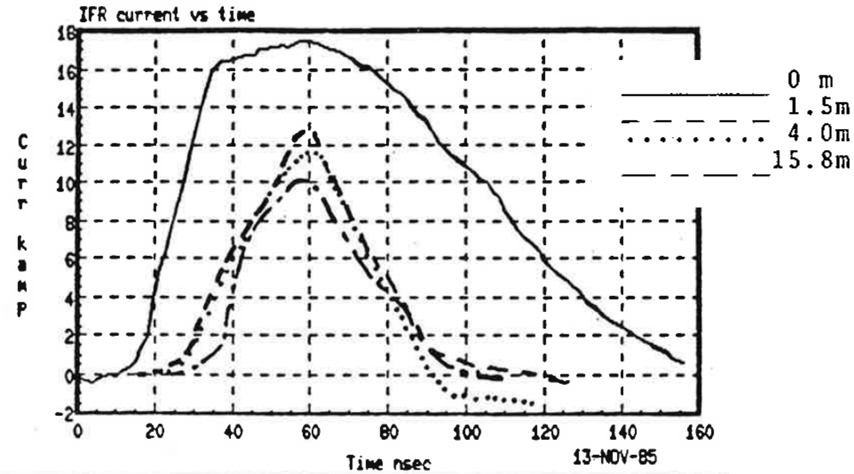
# Application: Beam propagation through plasma/gas; The time-resolved SCRAPE code was used for IFR propagation

- **Beam modelling**
  - Time resolved, non interacting disks
  - Gaussian radial density profile
  - Energy lost in propagation
  - Electrical neutralization by channel ions
  - Magnetic neutralization by channel return current
- **Channel modeling**
  - Stationary
  - Gaussian radial density profile
  - Axial gradients in density at entrance and exit
- **Emittance modeling included**
  - Entrance foil  scattering
  - Non-zero canonical angular momentum (immersed cathode)
  - Phase-mix damping

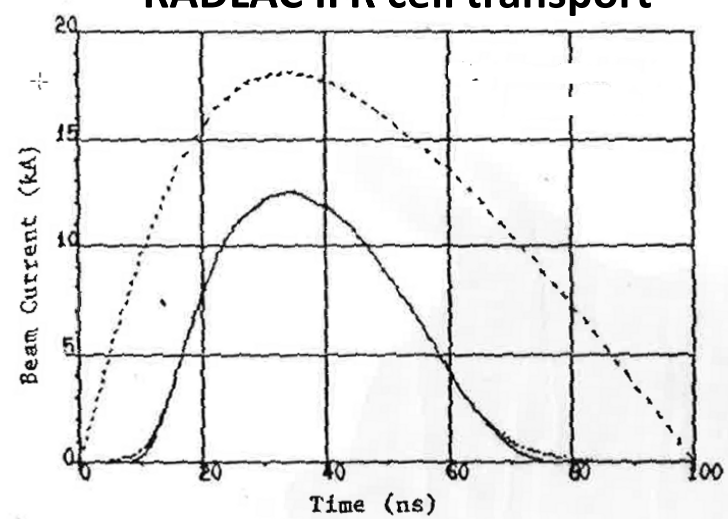
# Comparison of SCRAPE with experimental data from RADLAC ion focused transport (IFT) from LIA to back door (~ 16 m).

**RADLAC-II Experiment at Sandia:**

- **LIA: 18 kA, 20 MeV, 20 ns**
- **~16-m long tube**
- **10 – 100 uTorr argon gas**
- **Ion channel generated by tail-light bulb filaments, and confined by <50 G axial field**
- **Gaussian ion channel, FWHM ~ 1.6 cm, peak density ~ 1E12/cc**
- **<u>Also</u> simulations matched measured growth of rms beam radius from 0.8 cm at injection to ~5 cm at exit.**



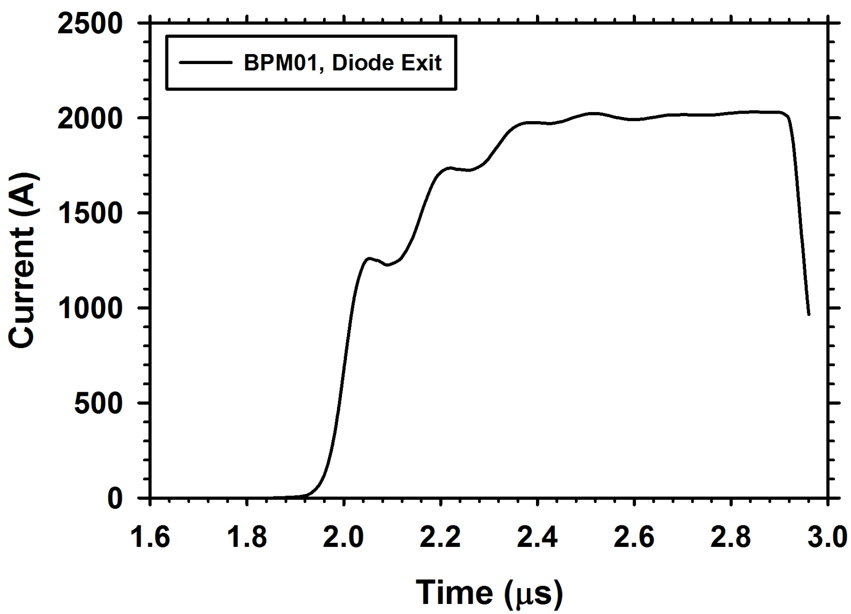

**RADLAC IFR cell transport**



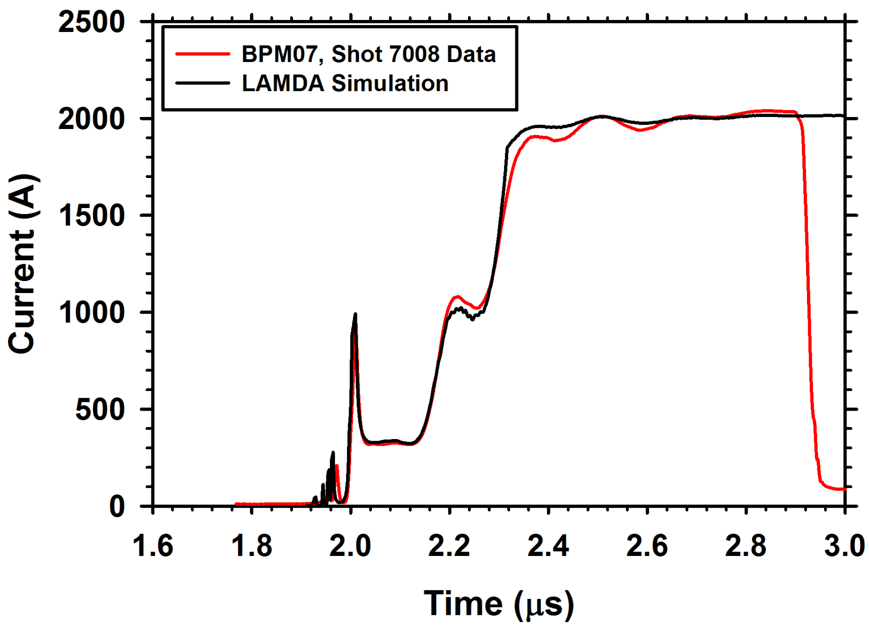

**SCRAPE simulation**

# Application: Time-resolved LAMDA simulations for predicting transport through DARHT-II BCUZ with different apertures installed.

**Beam current at exit of diode**




**Beam current at exit of BCUZ**




**This LAMDA simulation used measured current and voltage waveforms, actual beam tube and aperture geometry, and magnetic fields for the tune.**

## <u>Conclusion</u>: The envelope equation is a very useful tool for predicting high-current relativistic electron-beam behavior.

- The equation illuminates fundamental beam dynamics
  - Stable and unstable beam envelope osccillations
- Practical problems can be solved using the equation
  - Chromatic aberrations of solenoids
  - Space-charge limited transport in drift sections
  - Transport in gases
  - etc., etc.
- "Everyday" use at DARHT includes
  - Design of tunes
  - Prediction of off-normal beam behavior
    - E. g. beam spill due to failed magnets
  - Design of experiments
  - Interpretation of solenoid scan data